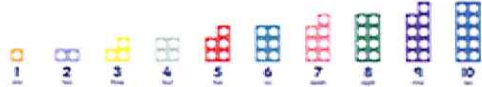



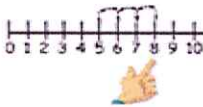




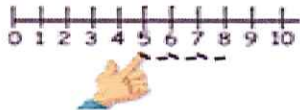
# Addition

Maths for young children should be meaningful. Where possible, concepts should be taught in the context of real life.

GUIDANCE / MODELS AND IMAGES	KEY VOCABULARY
<p>If available, Numicon shapes are introduced straight away and can be used to:</p> <ul style="list-style-type: none"> <li>• identify 1 more/less</li> <li>• combine pieces to add.</li> <li>• find number bonds.</li> <li>• add without counting.</li> </ul> <p>Children can record this by printing or drawing around Numicon pieces.</p>  <p>Children begin to combine groups of objects using concrete apparatus</p>  <p>Construct number sentences verbally or using cards to go with practical activities.</p> <p>Children are encouraged to read number sentences aloud in different ways          "Three add two equals 5" "5 is equal to three and two"</p> <p>Children make a record in pictures, words or symbols of addition activities already carried out.</p> <p>Solve simple problems using fingers</p>  <p><math>5 + 1 = 6</math></p> <p>Number tracks can be introduced to count up on and to find one more:</p>  <p>What is 1 more than 4? 1 more than 13?</p> <p>Number lines can then be used alongside number tracks and practical apparatus to solve addition calculations and word problems.</p> <p><math>5 + 3 = 8</math></p>  <p>Children will need opportunities to look at and talk about different models and images as they move between representations.</p>	<p>Games and songs can be a useful way to begin using vocabulary involved in addition e.g. Alice the Camel</p> <p>add</p> <p>more</p> <p>and</p> <p>make</p> <p>sum</p> <p>total</p> <p>altogether</p> <p>score</p> <p>double</p> <p>one more, two more, ten more...</p> <p>how many more to make...?</p> <p>how many more is... than...?</p>

# Subtraction

Maths for young children should be meaningful. Where possible, concepts should be taught in the context of real life.

GUIDANCE / MODELS AND IMAGES	KEY VOCABULARY
<p>Children begin with mostly pictorial representations</p> <p>XXX    <span style="border: 1px solid black; padding: 2px;">XX</span></p> <p>Concrete apparatus is used to relate subtraction to taking away and counting how many objects are left.</p> <p>Concrete apparatus models the subtraction of 2 objects from a set of 5.</p> <p>Construct number sentences verbally or using cards to go with practical activities.</p> <p>Children are encouraged to read number sentences aloud in different ways "five subtract one leaves four" "four is equal to five subtract one"</p> <p>Children make a record in pictures, words or symbols of subtraction activities already carried out.</p> <p>Solve simple problems using fingers</p> <p style="text-align: center;">    <math>5 - 1 = 4</math> </p> <p>Number tracks can be introduced to count back and to find one less:</p> <p style="text-align: center;"> <span style="border: 1px solid black; padding: 2px;">1</span> <span style="border: 1px solid black; padding: 2px;">2</span> <span style="border: 1px solid black; padding: 2px;">3</span> <span style="border: 1px solid black; padding: 2px;">4</span> <span style="border: 1px solid black; padding: 2px;">5</span> <span style="border: 1px solid black; padding: 2px;">6</span> </p> <p>What is 1 less than 9? 1 less than 20?</p> <p>Number lines can then be used alongside number tracks and practical apparatus to solve subtraction calculations and word problems. Children count back under the number line.</p> <p style="text-align: center;"> <math>8 - 3 = 5</math>  </p> <p>Children will need opportunities to look at and talk about different models and images as they move between representations.</p>	<p>Games and songs can be a useful way to begin using vocabulary involved in subtraction e.g.</p> <p>Five little men in a flying saucer</p> <p>take (away)</p> <p>leave</p> <p>how many are left/left over?</p> <p>how many have gone?</p> <p>one less, two less... ten less...</p> <p>how many fewer is... than...?</p> <p>difference between</p> <p>is the same as</p>

# Multiplication

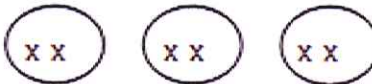
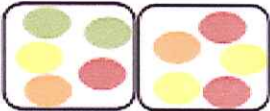
Maths for young children should be meaningful. Where possible, concepts should be taught in the context of real life.

GUIDANCE / MODELS AND IMAGES	KEY VOCABULARY
<p>The link between addition and multiplication can be introduced through doubling.</p> <p>If available, Numicon is used to visualise the repeated adding of the same number. These can then be drawn around or printed as a way of recording.</p> <p>Children begin with mostly pictorial representations:</p> <div data-bbox="392 662 772 742"> </div> <p>How many groups of 2 are there?</p> <p>Real life contexts and use of practical equipment to <u>count in repeated groups of the same size</u>:</p> <div data-bbox="392 901 929 997"> </div> <p>How many wheels are there altogether?</p> <div data-bbox="1075 917 1624 1021"> </div> <p>How much money do I have?</p> <div data-bbox="996 1117 1153 1204"> </div> <p>Count in twos; fives; tens both aloud and with objects</p> <p>Children are <u>given multiplication problems set in a real life context</u>. Children are encouraged to visualise the problem.</p> <p>How many fingers on two hands? How many sides on three triangles? How many legs on four ducks?</p> <p>Children are encouraged to read number sentences aloud in different ways "five times two makes ten" "ten is equal to five multiplied by two"</p>	<p>lots of</p> <p>groups of</p> <p>times</p> <p>multiply</p> <p>multiplied by</p> <p>multiple of</p> <p>once, twice, three times... ten times...</p> <p>...times as (big, long, wide... and so on)</p> <p>repeated addition</p> <p>double</p>

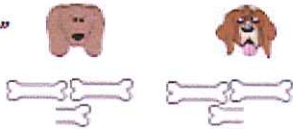


# Division and fractions

Maths for young children should be meaningful. Where possible, concepts should be taught in the context of real life.

GUIDANCE / MODELS AND IMAGES	KEY VOCABULARY
<p>The ELG states that children solve problems, including doubling, halving and sharing.</p> <p>Children need to see and hear representations of division as both grouping and sharing.</p> <p>Division can be introduced through halving.</p> <p>Children begin with mostly pictorial representations linked to real life contexts:</p> <div style="display: flex; align-items: flex-start;"> <div style="margin-right: 10px;">  </div> <div> <p><b>Grouping model</b> Mum has 6 socks. She grouped them into pairs – how many pairs did she make?</p> </div> </div> <div style="display: flex; align-items: flex-start; margin-top: 10px;"> <div style="margin-right: 10px;">  </div> <div> <p><b>Sharing model</b> I have 10 sweets. I want to share them with my friend. How many will we have each?</p> </div> </div> <p>Children have a go at recording the calculation that has been carried out.</p>	<p>halve</p> <p>share, share equally</p> <p>one each, two each, three each...</p> <p>group in pairs, threes...</p> <p>tens</p> <p>equal groups of</p> <p>divide</p> <p>divided by</p> <p>divided into</p> <p>left, left over</p>

## FRACTIONS

GUIDANCE / MODELS AND IMAGES	KEY VOCABULARY
<p>Although not explicit in the Development Matters document, the sharing model is a useful way of introducing young children to fractions and calculating with fractions.</p> <p>Setting the problems in real life context and solving them with <u>concrete apparatus</u> will support children's understanding.</p> <p>"I have got 5 bones to share between my two dogs. How many bones will they get each?"</p> <div style="display: flex; align-items: center; justify-content: center; margin-top: 10px;">  </div> <p>Children have a go at recording the calculation that has been carried out.</p> <p><math>2\frac{1}{2} + 2\frac{1}{2} = 5</math></p>	<p>As division vocabulary plus:</p> <p>fraction</p> <p>half</p> <p>halves</p> <p>third</p> <p>thirds</p>

# Addition

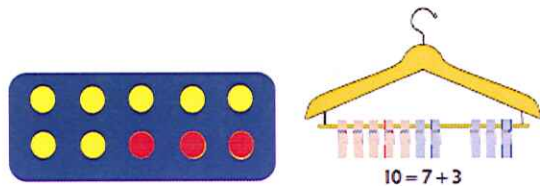
## Year 1

### Mental Strategies (addition and subtraction)

Children should experience regular counting on and back from different numbers in 1s and in multiples of 2, 5 and 10.

Children should memorise and reason with number bonds for numbers to 20, experiencing the = sign in different positions.

They should see addition and subtraction as related operations. E.g.  $7 + 3 = 10$  is related to  $10 - 3 = 7$ , understanding of which could be supported by an image like this.

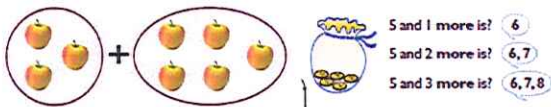


Use bundles of straws and Dienes to model partitioning teen numbers into tens and ones and develop understanding of place value.

Children have opportunities to explore partitioning numbers in different ways.

e.g.  $7 = 6 + 1$ ,  $7 = 5 + 2$ ,  $7 = 4 + 3 =$

Children should begin to understand addition as combining groups and counting on.



### Vocabulary

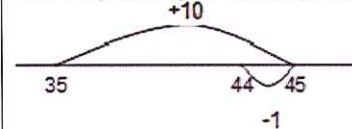
Addition, add, forwards, put together, more than, total, altogether, distance between, difference between, equals = same as, most, pattern, odd, even, digit, counting on.

## Year 2

### Mental Strategies

Children should count regularly, on and back, in steps of 2, 3, 5 and 10. Counting forwards in tens from any number should lead to adding multiples of 10.

Number lines should continue to be an important image to support mathematical thinking, for example to model how to add 9 by adding 10 and adjusting.



Children should practise addition to 20 to become increasingly fluent. They should use the facts they know to derive others, e.g. using  $7 + 3 = 10$  to find  $17 + 3 = 20$ ,  $70 + 30 = 100$

They should use concrete objects such as bead strings and number lines to explore missing numbers  $- 45 + \quad = 50$ .

As well as number lines, 100 squares could be used to explore patterns in calculations such as  $74 + 11$ ,  $77 + 9$  encouraging children to think about 'What do you notice?' where partitioning or adjusting is used.

Children should learn to check their calculations, by using the inverse.

They should continue to see addition as both combining groups and counting on.

They should use Dienes to model partitioning into tens and ones and learn to partition numbers in different ways e.g.  $23 = 20 + 3 = 10 + 13$ .

### Vocabulary

+, add, addition, more, plus, make, sum, total, altogether, how many more to make...? how many more is... than...? how much more is...? =, equals, sign, is the same as, Tens, ones, partition Near multiple of 10, tens boundary, More than, one more, two more... ten more... one hundred more

## Year 3

### Mental Strategies

Children should continue to count regularly, on and back, now including multiples of 4, 8, 50, and 100, and steps of 1/10.

The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged. This will help to develop children's understanding of working mentally.

Children should continue to partition numbers in different ways.

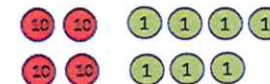
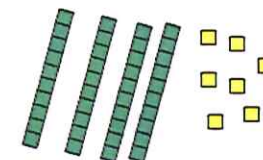
They should be encouraged to choose the mental strategies which are most efficient for the numbers involved, e.g.

Add the nearest multiple of 10, then adjust such as  $63 + 29$  is the same as  $63 + 30 - 1$ ;

counting on by partitioning the second number only such as  $72 + 31 = 72 + 30 + 1 = 102 + 1 = 103$

Manipulatives can be used to support mental imagery and conceptual understanding. Children need to be shown how these images are related eg.

What's the same? What's different?



### Vocabulary

Hundreds, tens, ones, estimate, partition, recombine, difference, decrease, near multiple of 10 and 100, inverse, rounding, column subtraction, exchange See also Y1 and Y2



### Generalisations

- True or false? Addition makes numbers bigger.
- True or false? You can add numbers in any order and still get the same answer.

(Links between addition and subtraction)

When introduced to the equals sign, children should see it as signifying equality. They should become used to seeing it in different positions.

Another example here...

### Some Key Questions

How many altogether? How many more to make...? I add ...more. What is the total? How many more is... than...?

How much more is...? One more, two more, ten more...

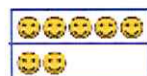
What can you see here?

Is this true or false?

What is the same? What is different?

### Generalisation

- Noticing what happens when you count in tens (the digits in the ones column stay the same)
- Odd + odd = even; odd + even = odd; etc
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems. This understanding could be supported by images such as this.



$$7 + ? = 10$$



### Some Key Questions

How many altogether? How many more to make...? How many more is... than...? How much more is...?

Is this true or false?

If I know that  $17 + 2 = 19$ , what else do I know? (e.g.  $2 + 17 = 19$ ;  $19 - 17 = 2$ ;  $19 - 2 = 17$ ;  $190 - 20 = 170$  etc).

What do you notice? What patterns can you see?

### Generalisations

Noticing what happens to the digits when you count in tens and hundreds.

Odd + odd = even etc (see Year 2)

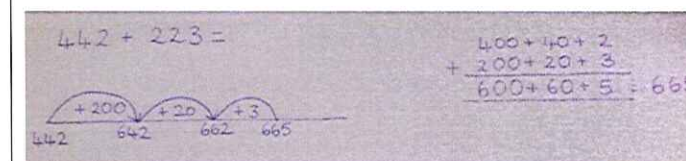
Inverses and related facts – develop fluency in finding related addition and subtraction facts.

Develop the knowledge that the inverse relationship can be used as a checking method.

### Key Questions

What do you notice? What patterns can you see?

When comparing two methods alongside each other: What's the same? What's different? Look at this number in the formal method; can you see where it is in the expanded method / on the number line?

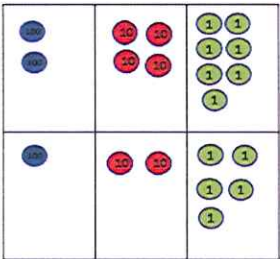
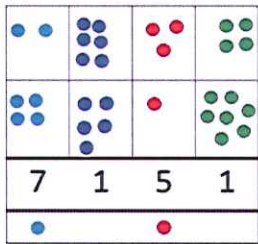


## Addition

Year 4	Year 5	Year 6
<p><b><u>Mental Strategies</u></b>  Children should continue to count regularly, on and back, now including multiples of 6, 7, 9, 25 and 1000, and steps of 1/100.  The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged where appropriate.  Children should continue to partition numbers in different ways.</p> <p>They should be encouraged to choose from a range of strategies:</p> <ul style="list-style-type: none"> <li>Counting forwards and backwards: <math>124 - 47</math>, count back 40 from 124, then 4 to 80, then 3 to 77</li> <li>Reordering: <math>28 + 75</math>, <math>75 + 28</math> (thinking of 28 as <math>25 + 3</math>)</li> <li>Partitioning: counting on or back: <math>5.6 + 3.7</math>, <math>5.6 + 3 + 0.7 = 8.6 + 0.7</math></li> <li>Partitioning: bridging through multiples of 10: <math>6070 - 4987</math>, <math>4987 + 13 + 1000 + 70</math></li> <li>Partitioning: compensating – <math>138 + 69</math>, <math>138 + 70 - 1</math></li> <li>Partitioning: using 'near' doubles - <math>160 + 170</math> is double 150, then add 10, then add 20, or double 160 and add 10, or double 170 and subtract 10</li> <li>Partitioning: bridging through 60 to calculate a time interval – What was the time 33 minutes before 2.15pm?</li> <li>Using known facts and place value to find related facts.</li> </ul> <p><b><u>Vocabulary</u></b>  add, addition, sum, more, plus, increase, sum, total, altogether, double, near double, how many more to make...? how much more? ones boundary, tens boundary, hundreds boundary, thousands boundary, tenths boundary, hundredths boundary, inverse, how many more/fewer? Equals sign, is the same as.</p>	<p><b><u>Mental Strategies</u></b>  Children should continue to count regularly, on and back, now including steps of powers of 10.  The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged where appropriate.  Children should continue to partition numbers in different ways.</p> <p>They should be encouraged to choose from a range of strategies:</p> <ul style="list-style-type: none"> <li>Counting forwards and backwards in tenths and hundredths: <math>1.7 + 0.55</math></li> <li>Reordering: <math>4.7 + 5.6 - 0.7</math>, <math>4.7 - 0.7 + 5.6 = 4 + 5.6</math></li> <li>Partitioning: counting on or back - <math>540 + 280</math>, <math>540 + 200 + 80</math></li> <li>Partitioning: bridging through multiples of 10:</li> <li>Partitioning: compensating: <math>5.7 + 3.9</math>, <math>5.7 + 4.0 - 0.1</math></li> <li>Partitioning: using 'near' double: <math>2.5 + 2.6</math> is double 2.5 and add 0.1 or double 2.6 and subtract 0.1</li> <li>Partitioning: bridging through 60 to calculate a time interval: It is 11.45. How many hours and minutes is it to 15.20?</li> <li>Using known facts and place value to find related facts.</li> </ul> <p><b><u>Vocabulary</u></b>  tens of thousands boundary,  Also see previous years</p> <p><b><u>Generalisation</u></b>  Sometimes, always or never true? The difference between a number and its reverse will be a multiple of 9.  What do you notice about the differences between consecutive square numbers?  <a href="#">Investigate <math>a - b = (a-1) - (b-1)</math> represented visually.</a></p> <p><b><u>Some Key Questions</u></b>  What do you notice?  What's the same? What's different?</p>	<p><b><u>Mental Strategies</u></b>  Consolidate previous years.</p> <p>Children should experiment with order of operations, investigating the effect of positioning the brackets in different places, e.g. <math>20 - 5 \times 3 = 5</math>; <math>(20 - 5) \times 3 = 45</math></p> <p><b><u>Vocabulary</u></b>  See previous years</p> <p><b><u>Generalisations</u></b>  Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as PEMDAS, or could be encouraged to design their own ways of remembering.  Sometimes, always or never true? Subtracting numbers makes them smaller.</p> <p><b><u>Some Key Questions</u></b>  What do you notice?  What's the same? What's different?  Can you convince me?  How do you know?</p>

<p><b><u>Generalisations</u></b> Investigate when re-ordering works as a strategy for subtraction. Eg. <math>20 - 3 - 10 = 20 - 10 - 3</math>, but <math>3 - 20 - 10</math> would give a different answer.</p> <p><b><u>Some Key Questions</u></b> What do you notice? What's the same? What's different? Can you convince me? How do you know?</p>	<p>Can you convince me? How do you know?</p>	
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Year 4	Year 5	Year 6
<p>Missing number/digit problems:</p> <p><b>Mental methods</b> should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving.</p> <p><b>Written methods (progressing to 4-digits)</b> Expanded column addition modelled with place value counters, progressing to calculations with 4-digit numbers.</p> <div data-bbox="255 555 748 826">  <math display="block">\begin{array}{r} 200 + 40 + 7 \\ 100 + 20 + 5 \\ 300 + 60 + 12 = 372 \end{array}</math> <math display="block">\begin{array}{r} 247 \\ +125 \\ \hline 12 \\ 60 \\ 300 \\ \hline 372 \end{array}</math> </div> <p><b>Compact written method</b> Extend to numbers with at least four digits.</p> <div data-bbox="286 911 672 1155">  <math display="block">\begin{array}{r} 2634 \\ +4517 \\ \hline 7151 \end{array}</math> </div> <p>Children should be able to make the choice of reverting to expanded methods if experiencing any difficulty.</p> <p>Extend to up to two places of decimals (same number of decimals places) and adding several numbers (with different numbers of digits).</p> $\begin{array}{r} 72.8 \\ + 54.6 \\ \hline 127.4 \\ 1 \quad 1 \end{array}$	<p>Missing number/digit problems:</p> <p><b>Mental methods</b> should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving. Children should practise with increasingly large numbers to aid fluency e.g. <math>12462 + 2300 = 14762</math></p> <p><b>Written methods (progressing to more than 4-digits)</b> As year 4, progressing when understanding of the expanded method is secure, children will move on to the formal columnar method for whole numbers and decimal numbers as an efficient written algorithm.</p> $\begin{array}{r} 172.83 \\ + 54.68 \\ \hline 227.51 \\ 1 \quad 1 \quad 1 \end{array}$ <p>Place value counters can be used alongside the columnar method to develop understanding of addition with decimal numbers.</p>	<p>Missing number/digit problems:</p> <p><b>Mental methods</b> should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving.</p> <p><b>Written methods</b> As year 5, progressing to larger numbers, aiming for both conceptual understanding and procedural fluency with columnar method to be secured. Continue calculating with decimals, including those with different numbers of decimal places</p> <p><b>Problem Solving</b> Teachers should ensure that pupils have the opportunity to apply their knowledge in a variety of contexts and problems (exploring cross curricular links) to deepen their understanding.</p>

## Year 1

### + = signs and missing numbers

Children need to understand the concept of equality before using the '=' sign. Calculations should be written either side of the equality sign so that the sign is not just interpreted as 'the answer'.

$$2 = 1 + 1$$

$$2 + 3 = 4 + 1$$

Missing numbers need to be placed in all possible places.

$$3 + 4 = \square$$

$$\square = 3 + 4$$

$$3 + \square = 7$$

$$7 = \square + 4$$

### Counting and Combining sets of Objects

Combining two sets of objects (aggregation) which will progress onto adding on to a set (augmentation)

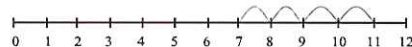


### Understanding of counting on with a numbertrack.



Understanding of counting on with a numberline  
(supported by models and images).

$$7 + 4$$



## Year 2

Missing number problems e.g.  $14 + 5 = 10 + \square$   $32 + \square + \square = 100$   
 $35 = 1 + \square + 5$

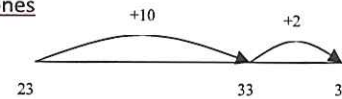
It is valuable to use a range of representations (also see Y1). Continue to use numberlines to develop understanding of:

### Counting on in tens and ones

$$23 + 12 = 23 + 10 + 2$$

$$= 33 + 2$$

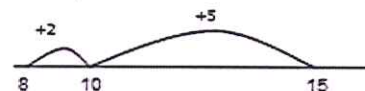
$$= 35$$



### Partitioning and bridging through 10.

The steps in addition often bridge through a multiple of 10 e.g. Children should be able to partition the 7 to relate adding the 2 and then the 5.

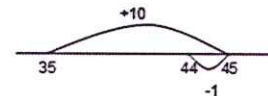
$$8 + 7 = 15$$



### Adding 9 or 11 by adding 10 and adjusting by 1

e.g. Add 9 by adding 10 and adjusting by 1

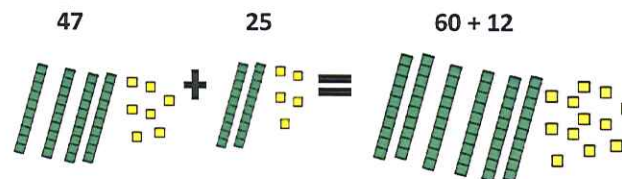
$$35 + 9 = 44$$



### Towards a Written Method

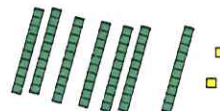
Partitioning in different ways and recombine

$$47 + 25$$



Leading to exchanging:

$$72$$



### Expanded written method

$$40 + 7 + 20 + 5 =$$

$$40 + 20 + 7 + 5 =$$

$$60 + 12 = 72$$

$$\begin{array}{r} 40 + 7 \\ + 20 + 5 \\ \hline 60 + 12 = 72 \end{array}$$

## Year 3

Missing number problems using a range of equations as in Year 1 and 2 but with appropriate, larger numbers.

### Partition into tens and ones

Partition both numbers and recombine.

Count on by partitioning the second number only e.g.

$$247 + 125 = 247 + 100 + 20 + 5$$

$$= 347 + 20 + 5$$

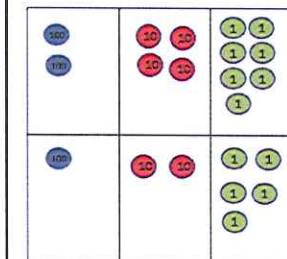
$$= 367 + 5$$

$$= 372$$

Children need to be secure adding multiples of 100 and 10 to any three-digit number including those that are not multiples of 10.

### Towards a Written Method

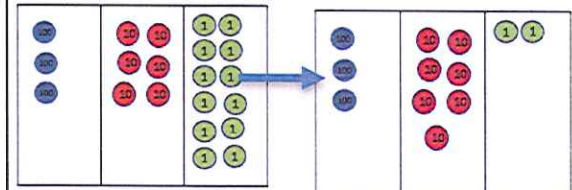
Introduce expanded column addition modelled with place value counters (Dienes could be used for those who need a less abstract representation)



$$\begin{array}{r} 200 + 40 + 7 \\ 100 + 20 + 5 \\ \hline 300 + 60 + 12 = 372 \end{array}$$

$$\begin{array}{r} 247 \\ + 125 \\ \hline 12 \\ 60 \\ 300 \\ \hline 372 \end{array}$$

Leading to children understanding the exchange between tens and ones.



Some children may begin to use a formal columnar algorithm, initially introduced alongside the expanded method. The formal method should be seen as a more streamlined version of the expanded method, not a new method.

$$\begin{array}{r} 247 \\ + 125 \\ \hline 372 \\ 10 \end{array}$$



# Subtraction

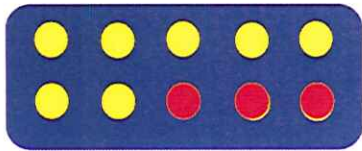
## Year 1

### Mental Strategies

Children should experience regular counting on and back from different numbers in 1s and in multiples of 2, 5 and 10.

Children should memorise and reason with number bonds for numbers to 20, experiencing the = sign in different positions.

They should see addition and subtraction as related operations. E.g.  $7 + 3 = 10$  is related to  $10 - 3 = 7$ , understanding of which could be supported by an image like this.

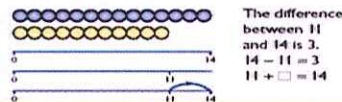


Use bundles of straws and Dienes to model partitioning teen numbers into tens and ones.

Children should begin to understand subtraction as both taking away and finding the difference between, and should find small differences by counting on.



Subtraction as "taking away"



Subtraction as "the difference between"

### Vocabulary

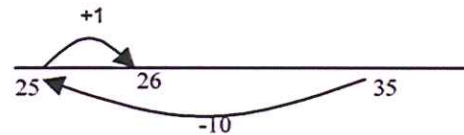
Subtraction, subtract, take away, distance between, difference between, more than, minus, less than, equals = same as, most, least, pattern, odd, even, digit,

## Year 2

### Mental Strategies

Children should count regularly, on and back, in steps of 2, 3, 5 and 10. Counting back in tens from any number should lead to subtracting multiples of 10.

Number lines should continue to be an important image to support thinking, for example to model how to subtract 9 by adjusting.



Children should practise subtraction to 20 to become increasingly fluent. They should use the facts they know to derive others, e.g. using  $10 - 7 = 3$  and  $7 = 10 - 3$  to calculate  $100 - 70 = 30$  and  $70 = 100 - 30$ .

91	92	93	94	95	96	97	98	99	100
81	82	83	84	85	86	87	88	89	90
71	72	73	74	75	76	77	78	79	80
61	62	63	64	65	66	67	68	69	70
51	52	53	54	55	56	57	58	59	60
41	42	43	44	45	46	47	48	49	50
31	32	33	34	35	36	37	38	39	40
21	22	23	24	25	26	27	28	29	30
11	12	13	14	15	16	17	18	19	20
1	2	3	4	5	6	7	8	9	10

As well as number lines, 100 squares could be used to model calculations such as  $74 - 11$ ,  $77 - 9$  or  $36 - 14$ , where partitioning or adjusting are used. On the example above, 1 is in the bottom left corner so that 'up' equates to 'add'.

Children should learn to check their calculations, including by adding to check.

They should continue to see subtraction as both take away and finding the difference, and should find a small difference by counting up.

They should use Dienes to model partitioning into tens and ones and learn to partition numbers in different ways e.g.  $23 = 20 + 3 = 10 + 13$ .

## Year 3

### Mental Strategies

Children should continue to count regularly, on and back, now including multiples of 4, 8, 50, and 100, and steps of 1/10.

The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged.

Children should continue to partition numbers in difference ways.

They should be encouraged to choose the mental strategies which are most efficient for the numbers involved, e.g. counting up (difference, or complementary addition) for  $201 - 198$ ; counting back (taking away / partition into tens and ones) for  $201 - 12$ .

Calculators can usefully be introduced to encourage fluency by using them for games such as 'Zap' [e.g. Enter the number 567. Can you 'zap' the 6 digit and make the display say 507 by subtracting 1 number?]

The strategy of adjusting can be taken further, e.g. subtract 100 and add one back on to subtract 99. Subtract other near multiples of 10 using this strategy.

### Vocabulary

Hundreds, tens, ones, estimate, partition, recombine, difference, decrease, near multiple of 10 and 100, inverse, rounding, column subtraction, exchange  
See also Y1 and Y2

### Generalisations

Noticing what happens to the digits when you count in tens and hundreds.

Odd – odd = even etc (see Year 2)

Inverses and related facts – develop fluency in finding related addition and subtraction facts.

Develop the knowledge that the inverse relationship can be used as a checking method.

### Key Questions

What do you notice? What patterns can you see?



### Generalisations

- True or false? Subtraction makes numbers smaller
- When introduced to the equals sign, children should see it as signifying equality. They should become used to seeing it in different positions.

Children could see the image below and consider, "What can you see here?" e.g.

3 yellow, 1 red, 1 blue.  $3 + 1 + 1 = 5$

2 circles, 2 triangles, 1

square.  $2 + 2 + 1 = 5$

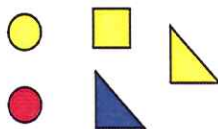
I see 2 shapes with

curved lines and 3 with

straight lines.  $5 = 2 + 3$

$5 = 3 + 1 + 1 = 2 + 2 + 1 =$

$2 + 3$



### Some Key Questions

How many more to make...? How many more is... than...?

How much more is...? How many are left/left over? How

many have gone? One less, two less, ten less... How

many fewer is... than...? How much less is...?

What can you see here?

Is this true or false?

### Vocabulary

Subtraction, subtract, take away, difference, difference between, minus

Tens, ones, partition

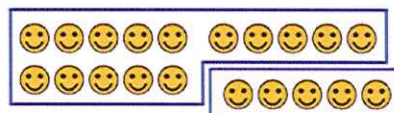
Near multiple of 10, tens boundary

Less than, one less, two less... ten less... one hundred less

More, one more, two more... ten more... one hundred more

### Generalisation

- Noticing what happens when you count in tens (the digits in the ones column stay the same)
- Odd – odd = even; odd – even = odd; etc
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems. This understanding could be supported by images such as this.



$$15 + 5 = 20$$

### Some Key Questions

How many more to make...? How many more is... than...? How

much more is...? How many are left/left over? How many

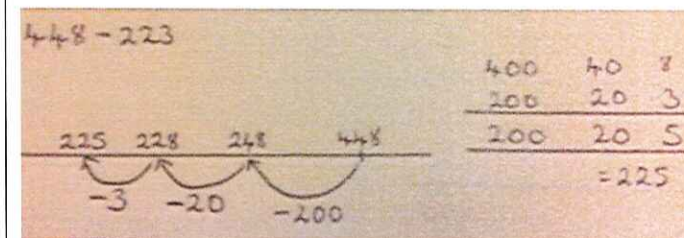
fewer is... than...? How much less is...?

Is this true or false?

If I know that  $7 + 2 = 9$ , what else do I know? (e.g.  $2 + 7 = 9$ ;  $9 - 7 = 2$ ;  $9 - 2 = 7$ ;  $90 - 20 = 70$  etc).

What do you notice? What patterns can you see?

When comparing two methods alongside each other: What's the same? What's different? Look at this number in the formal method; can you see where it is in the expanded method / on the number line



## Subtraction

Year 4	Year 5	Year 6
<p><b><u>Mental Strategies</u></b> Children should continue to count regularly, on and back, now including multiples of 6, 7, 9, 25 and 1000, and steps of 1/100. The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged where appropriate. Children should continue to partition numbers in different ways.</p> <p>They should be encouraged to choose from a range of strategies:</p> <ul style="list-style-type: none"> <li>Counting forwards and backwards: <math>124 - 47</math>, count back 40 from 124, then 4 to 80, then 3 to 77</li> <li>Reordering: <math>28 + 75</math>, <math>75 + 28</math> (thinking of 28 as <math>25 + 3</math>)</li> <li>Partitioning: counting on or back: <math>5.6 + 3.7</math>, <math>5.6 + 3 + 0.7 = 8.6 + 0.7</math></li> <li>Partitioning: bridging through multiples of 10: <math>6070 - 4987</math>, <math>4987 + 13 + 1000 + 70</math></li> <li>Partitioning: compensating - <math>138 + 69</math>, <math>138 + 70 - 1</math></li> <li>Partitioning: using 'near' doubles - <math>160 + 170</math> is double 150, then add 10, then add 20, or double 160 and add 10, or double 170 and subtract 10</li> <li>Partitioning: bridging through 60 to calculate a time interval - What was the time 33 minutes before 2.15pm?</li> <li>Using known facts and place value to find related facts.</li> </ul> <p><b><u>Vocabulary</u></b> add, addition, sum, more, plus, increase, sum, total, altogether, double, near double, how many more to make...? how much more? ones boundary, tens boundary, hundreds boundary, thousands boundary, tenths boundary, hundredths boundary, inverse, how many more/fewer? Equals sign, is the same as.</p>	<p><b><u>Mental Strategies</u></b> Children should continue to count regularly, on and back, now including steps of powers of 10. The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged where appropriate. Children should continue to partition numbers in different ways.</p> <p>They should be encouraged to choose from a range of strategies:</p> <ul style="list-style-type: none"> <li>Counting forwards and backwards in tenths and hundredths: <math>1.7 + 0.55</math></li> <li>Reordering: <math>4.7 + 5.6 - 0.7</math>, <math>4.7 - 0.7 + 5.6 = 4 + 5.6</math></li> <li>Partitioning: counting on or back - <math>540 + 280</math>, <math>540 + 200 + 80</math></li> <li>Partitioning: bridging through multiples of 10:</li> <li>Partitioning: compensating: <math>5.7 + 3.9</math>, <math>5.7 + 4.0 - 0.1</math></li> <li>Partitioning: using 'near' double: <math>2.5 + 2.6</math> is double 2.5 and add 0.1 or double 2.6 and subtract 0.1</li> <li>Partitioning: bridging through 60 to calculate a time interval: It is 11.45. How many hours and minutes is it to 15.20?</li> <li>Using known facts and place value to find related facts.</li> </ul> <p><b><u>Vocabulary</u></b> tens of thousands boundary, Also see previous years</p> <p><b><u>Generalisation</u></b> Sometimes, always or never true? The difference between a number and its reverse will be a multiple of 9. What do you notice about the differences between consecutive square numbers? <a href="#">Investigate <math>a - b = (a-1) - (b-1)</math> represented visually.</a></p> <p><b><u>Some Key Questions</u></b> What do you notice? What's the same? What's different?</p>	<p><b><u>Mental Strategies</u></b> Consolidate previous years.</p> <p>Children should experiment with order of operations, investigating the effect of positioning the brackets in different places, e.g. <math>20 - 5 \times 3 = 5</math>; <math>(20 - 5) \times 3 = 45</math></p> <p><b><u>Vocabulary</u></b> See previous years</p> <p><b><u>Generalisations</u></b> Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as PEMDAS, or could be encouraged to design their own ways of remembering. Sometimes, always or never true? Subtracting numbers makes them smaller.</p> <p><b><u>Some Key Questions</u></b> What do you notice? What's the same? What's different? Can you convince me? How do you know?</p>

<p><b><u>Generalisations</u></b> Investigate when re-ordering works as a strategy for subtraction. Eg. <math>20 - 3 - 10 = 20 - 10 - 3</math>, but <math>3 - 20 - 10</math> would give a different answer.</p> <p><b><u>Some Key Questions</u></b> What do you notice? What's the same? What's different? Can you convince me? How do you know?</p>	<p>Can you convince me? How do you know?</p>	
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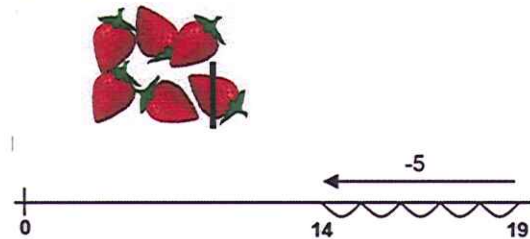


## Year 1

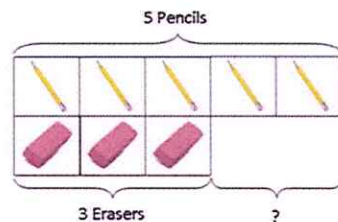
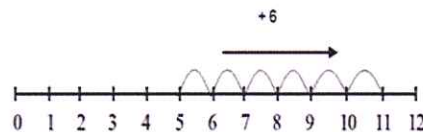
Missing number problems e.g.  $7 = \square - 9$ ;  $20 - \square = 9$ ;  $15 - 9 = \square$ ;  $\square - \square = 11$ ;  $16 - 0 = \square$

Use concrete objects and pictorial representations. If appropriate, progress from using number lines with every number shown to number lines with significant numbers shown.

Understand subtraction as take-away:



Understand subtraction as finding the difference:



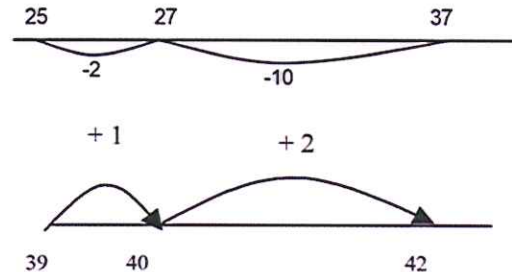
The above model would be introduced with concrete objects which children can move (including cards with pictures) before progressing to pictorial representation.

The use of other images is also valuable for modelling subtraction e.g. Numicon, bundles of straws, Dienes apparatus, multi-link cubes, bead strings

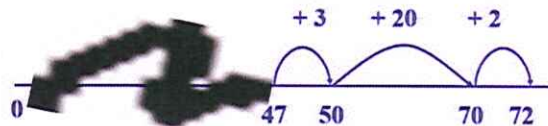
## Year 2

Missing number problems e.g.  $52 - 8 = \square$ ;  $\square - 20 = 25$ ;  $22 = \square - 21$ ;  $6 + \square + 3 = 11$

It is valuable to use a range of representations (also see Y1). Continue to use number lines to model take-away and difference. E.g.



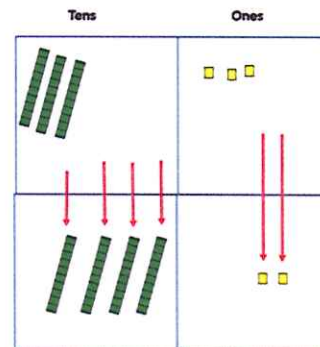
The link between the two may be supported by an image like this, with 47 being taken away from 72, leaving the difference, which is 25.



The bar model should continue to be used, as well as images in the context of measures.

### Towards written methods

Recording addition and subtraction in expanded columns can support understanding of the quantity aspect of place value and prepare for efficient written methods with larger numbers. The numbers may be represented with Dienes apparatus. E.g.  $75 - 42$



$$\begin{array}{r} 70 \\ - 40 \\ \hline 30 \end{array} \begin{array}{r} 5 \\ 2 \\ \hline 3 \end{array}$$

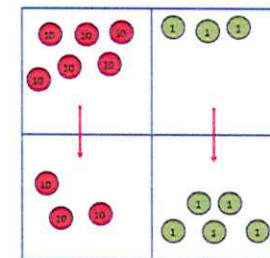
## Year 3

Missing number problems e.g.  $\square = 43 - 27$ ;  $145 - \square = 138$ ;  $274 - 30 = \square$ ;  $245 - \square = 195$ ;  $532 - 200 = \square$ ;  $364 - 153 = \square$

**Mental methods** should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving (see Y1 and Y2). Children should make choices about whether to use complementary addition or counting back, depending on the numbers involved.

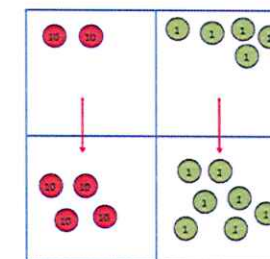
### Written methods (progressing to 3-digits)

Introduce expanded column subtraction with no decomposition, modelled with place value counters (Dienes could be used for those who need a less abstract representation)



$$\begin{array}{r} 90 \ 8 \\ - 30 \ 5 \\ \hline 60 \ 3 \end{array}$$

For some children this will lead to exchanging, modelled using place value counters (or Dienes).



$$\begin{array}{r} 70 \ 2 \\ - 40 \ 7 \\ \hline 20 \ 5 \end{array}$$


A number line and expanded column method may be compared next to each other.

Some children may begin to use a formal columnar algorithm, initially introduced alongside the expanded method. The formal method should be seen as a more streamlined version of the expanded method, not a new method.

Year 4	Year 5	Year 6
<p>Missing number/digit problems: <math>456 + \square = 710</math>; <math>1\square7 + 6\square = 200</math>; <math>60 + 99 + \square = 340</math>; <math>200 - 90 - 80 = \square</math>; <math>225 - \square = 150</math>; <math>\square - 25 = 67</math>; <math>3450 - 1000 = \square</math>; <math>\square - 2000 = 900</math></p> <p><b>Mental methods</b> should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving.</p> <p><b>Written methods (progressing to 4-digits)</b></p> <p>Expanded column subtraction with decomposition, modelled with place value counters, progressing to calculations with 4-digit numbers.</p> <div data-bbox="271 644 795 932"> <math display="block">\begin{array}{r} 200\overset{2}{3}2 \\ - 100\overset{1}{1}04 \\ \hline 100\overset{1}{1}08 \end{array}</math> </div> <p>If understanding of the expanded method is secure, children will move on to the formal method of decomposition, which again can be initially modelled with place value counters.</p> <div data-bbox="253 1136 685 1449"> <math display="block">\begin{array}{r} 232 \\ - 114 \\ \hline 118 \end{array}</math> </div>	<p>Missing number/digit problems: <math>6.45 = 6 + 0.4 + \square</math>; <math>119 - \square = 86</math>; <math>1\,000\,000 - \square = 999\,000</math>; <math>600\,000 + \square + 1000 = 671\,000</math>; <math>12\,462 - 2\,300 = \square</math></p> <p><b>Mental methods</b> should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving.</p> <p><b>Written methods (progressing to more than 4-digits)</b></p> <p>When understanding of the expanded method is secure, children will move on to the formal method of decomposition, which can be initially modelled with place value counters.</p> <div data-bbox="846 657 1467 981"> <math display="block">\begin{array}{r} 6232 \\ - 4814 \\ \hline 1418 \end{array}</math> </div> <p>Progress to calculating with decimals, including those with different numbers of decimal places.</p>	<p>Missing number/digit problems: <math>\square</math> and <math>\#</math> each stand for a different number. <math>\# = 34</math>. <math>\# + \# = \square + \square + \#</math>. What is the value of <math>\square</math>? What if <math>\# = 28</math>? What if <math>\# = 21</math></p> <p><math>10\,000\,000 = 9\,000\,100 + \square</math></p> <p><math>7 - 2 \times 3 = \square</math>; <math>(7 - 2) \times 3 = \square</math>; <math>(\square - 2) \times 3 = 15</math></p> <p><b>Mental methods</b> should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving.</p> <p><b>Written methods</b></p> <p>As year 5, progressing to larger numbers, aiming for both conceptual understanding and procedural fluency with decomposition to be secured.</p> <p>Teachers may also choose to introduce children to other efficient written layouts which help develop conceptual understanding. For example:</p> <div data-bbox="1720 858 1800 1173"> <math display="block">\begin{array}{r} 326 \\ - 148 \\ - 2 \\ - 20 \\ \hline 200 \\ \hline 178 \end{array}</math> </div> <p>Continue calculating with decimals, including those with different numbers of decimal places.</p>



# Multiplication

Year 1	Year 2	Year 3
<p><b><u>Mental Strategies</u></b>  Children should experience <u>regular counting</u> on and back from different numbers in 1s and in multiples of 2, 5 and 10.  Children should memorise and reason with numbers in 2, 5 and 10 times tables  They should see ways to represent odd and even numbers. This will help them to understand the pattern in numbers.</p>  <p>Children should begin to understand multiplication as scaling in terms of double and half. (e.g. that tower of cubes is double the height of the other tower)</p> <p><b><u>Vocabulary</u></b>  Ones, groups, lots of, doubling  repeated addition  groups of, lots of, times, columns, rows  longer, bigger, higher etc  times as (big, long, wide ...etc)</p> <p><b><u>Generalisations</u></b>  Understand 6 counters can be arranged as 3+3 or 2+2+2</p> <p>Understand that when counting in twos, the numbers are always even.</p> <p><b><u>Some Key Questions</u></b>  Why is an even number an even number?  What do you notice?  What's the same? What's different?  Can you convince me?  How do you know?</p>	<p><b><u>Mental Strategies</u></b>  Children should count regularly, on and back, in steps of 2, 3, 5 and 10.  Number lines should continue to be an important image to support thinking, for example</p> <p>Children should practise times table facts  <math>2 \times 1 =</math>  <math>2 \times 2 =</math>  <math>2 \times 3 =</math></p> <p>Use a clock face to support understanding of counting in 5s.  Use money to support counting in 2s, 5s, 10s, 20s, 50s</p> <p><b><u>Vocabulary</u></b>  multiple, multiplication array, multiplication tables / facts  groups of, lots of, times, columns, rows</p> <p><b><u>Generalisation</u></b>  Commutative law shown on array (video)</p> <p>Repeated addition can be shown mentally on a number line</p> <p>Inverse relationship between multiplication and division. Use an array to explore how numbers can be organised into groups.</p> <p><b><u>Some Key Questions</u></b>  What do you notice?  What's the same? What's different?  Can you convince me?  How do you know?</p>	<p><b><u>Mental Strategies</u></b>  Children should continue to count regularly, on and back, now including multiples of 4, 8, 50, and 100, and steps of 1/10.  The number line should continue to be used as an important image to support thinking, and the use of informal jottings and drawings to solve problems should be encouraged.</p> <p>Children should practise times table facts  <math>3 \times 1 =</math>  <math>3 \times 2 =</math>  <math>3 \times 3 =</math></p> <p><b><u>Vocabulary</u></b>  partition  grid method  inverse</p> <p><b><u>Generalisations</u></b>  Connecting x2, x4 and x8 through multiplication facts</p> <p>Comparing times tables with the same times tables which is ten times bigger. If <math>4 \times 3 = 12</math>, then we know <math>4 \times 30 = 120</math>. Use place value counters to demonstrate this.</p> <p>When they know multiplication facts up to x12, do they know what x13 is? (i.e. can they use 4x12 to work out 4x13 and 4x14 and beyond?)</p> <p><b><u>Some Key Questions</u></b>  What do you notice?  What's the same? What's different?  Can you convince me?  How do you know?</p>



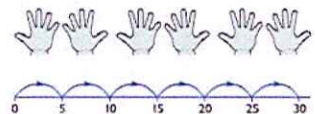
## Multiplication

Year 4	Year 5	Year 6
<p><b><u>Mental Strategies</u></b>  Children should continue to count regularly, on and back, now including multiples of 6, 7, 9, 25 and 1000, and steps of 1/100.  Become fluent and confident to recall all tables to x 12  Use the context of a week and a calendar to support the 7 times table (e.g. how many days in 5 weeks?)  Use of finger strategy for 9 times table.</p> <p>Multiply 3 numbers together  The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged.  They should be encouraged to choose from a range of strategies:</p> <ul style="list-style-type: none"> <li>- Partitioning using x10, x20 etc</li> <li>- Doubling to solve x2, x4, x8</li> <li>- Recall of times tables</li> <li>- Use of commutativity of multiplication</li> </ul> <p><b><u>Vocabulary</u></b>  Factor</p> <p><b><u>Generalisations</u></b>  Children given the opportunity to investigate numbers multiplied by 1 and 0.</p> <p>When they know multiplication facts up to x12, do they know what x13 is? (i.e. can they use 4x12 to work out 4x13 and 4x14 and beyond?)</p> <p><b><u>Some Key Questions</u></b>  What do you notice?  What's the same? What's different?  Can you convince me?  How do you know?</p>	<p><b><u>Mental Strategies</u></b>  Children should continue to count regularly, on and back, now including steps of powers of 10.  Multiply by 10, 100, 1000, including decimals (Moving Digits ITP)  The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged.  They should be encouraged to choose from a range of strategies to solve problems mentally:</p> <ul style="list-style-type: none"> <li>- Partitioning using x10, x20 etc</li> <li>- Doubling to solve x2, x4, x8</li> <li>- Recall of times tables</li> <li>- Use of commutativity of multiplication</li> </ul> <p>If children know the times table facts to 12 x 12. Can they use this to recite other times tables (e.g. the 13 times tables or the 24 times table)</p> <p><b><u>Vocabulary</u></b>  cube numbers  prime numbers  square numbers  common factors  prime number, prime factors  composite numbers</p> <p><b><u>Generalisation</u></b>  Relating arrays to an understanding of square numbers and making cubes to show cube numbers.  Understanding that the use of scaling by multiples of 10 can be used to convert between units of measure (e.g. metres to kilometres means to times by 1000)</p> <p><b><u>Some Key Questions</u></b>  What do you notice?  What's the same? What's different?  Can you convince me?  How do you know?</p>	<p><b><u>Mental Strategies</u></b>  Consolidate previous years.</p> <p>Children should experiment with order of operations, investigating the effect of positioning the brackets in different places, e.g. <math>20 - 5 \times 3 = 5</math>; <math>(20 - 5) \times 3 = 45</math></p> <p>They should be encouraged to choose from a range of strategies to solve problems mentally:</p> <ul style="list-style-type: none"> <li>- Partitioning using x10, x20 etc</li> <li>- Doubling to solve x2, x4, x8</li> <li>- Recall of times tables</li> <li>- Use of commutativity of multiplication</li> </ul> <p>If children know the times table facts to 12 x 12. Can they use this to recite other times tables (e.g. the 13 times tables or the 24 times table)</p> <p><b><u>Vocabulary</u></b>  See previous years  common factor</p> <p><b><u>Generalisations</u></b>  Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as PEMDAS, or could be encouraged to design their own ways of remembering.  Understanding the use of multiplication to support conversions between units of measurement.</p> <p><b><u>Some Key Questions</u></b>  What do you notice?  What's the same? What's different?  Can you convince me?  How do you know?</p>

## Year 1

Understand multiplication is related to doubling and combining groups of the same size (repeated addition)

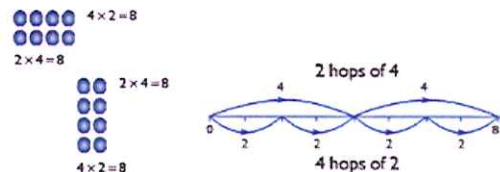
Washing line, and other practical resources for counting. Concrete objects. Numicon; bundles of straws, bead strings



Problem solving with concrete objects (including money and measures)

Use cuisenaire and bar method to develop the vocabulary relating to 'times' – Pick up five, 4 times

Use arrays to understand multiplication can be done in any order (commutative)



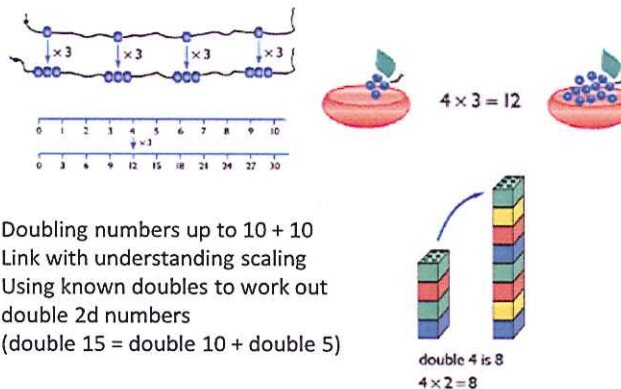
## Year 2

Expressing multiplication as a number sentence using  $\times$   
 Using understanding of the inverse and practical resources to solve missing number problems.

$$\begin{aligned} 7 \times 2 &= \square & \square &= 2 \times 7 \\ 7 \times \square &= 14 & 14 &= \square \times 7 \\ \square \times 2 &= 14 & 14 &= 2 \times \square \\ \square \times \square &= 14 & 14 &= \square \times \square \end{aligned}$$

Develop understanding of multiplication using array and number lines (see Year 1). Include multiplications not in the 2, 5 or 10 times tables.

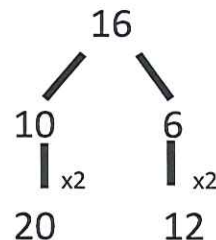
Begin to develop understanding of multiplication as scaling (3 times bigger/taller)



Doubling numbers up to  $10 + 10$   
 Link with understanding scaling  
 Using known doubles to work out double 2d numbers  
 (double 15 = double 10 + double 5)

### Towards written methods

Use jottings to develop an understanding of doubling two digit numbers.



## Year 3

Missing number problems  
 Continue with a range of equations as in Year 2 but with appropriate numbers.

### Mental methods

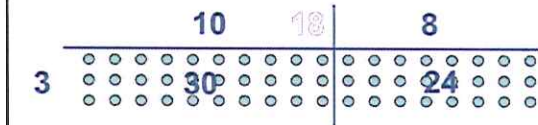
Doubling 2 digit numbers using partitioning

Demonstrating multiplication on a number line – jumping in larger groups of amounts

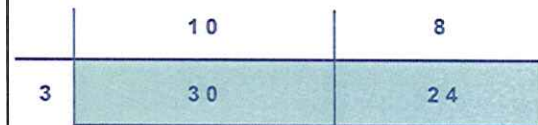
$$13 \times 4 = 10 \text{ groups } 4 = 3 \text{ groups of } 4$$

### Written methods (progressing to 2d x 1d)

Developing written methods using understanding of visual images



Develop onto the grid method



Give children opportunities for children to explore this and deepen understanding using Dienes apparatus and place value counters



## Year 4

Continue with a range of equations as in Year 2 but with appropriate numbers. Also include equations with missing digits

$$\square 2 \times 5 = 160$$

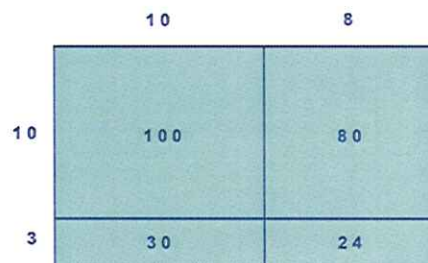
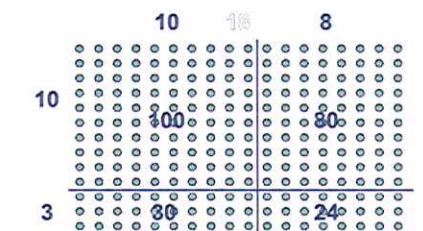
### Mental methods

Counting in multiples of 6, 7, 9, 25 and 1000, and steps of  $1/100$ .

Solving practical problems where children need to scale up. Relate to known number facts. (e.g. how tall would a 25cm sunflower be if it grew 6 times taller?)

### Written methods (progressing to 3d x 2d)

Children to embed and deepen their understanding of the grid method to multiply  $2d \times 2d$ . Ensure this is still linked back to their understanding of arrays and place value counters.



## Year 5

Continue with a range of equations as in Year 2 but with appropriate numbers. Also include equations with missing digits

### Mental methods

X by 10, 100, 1000 using moving digits ITP

Use practical resources and jottings to explore equivalent statements (e.g.  $4 \times 35 = 2 \times 2 \times 35$ )

Recall of prime numbers up to 19 and identify prime numbers up to 100 (with reasoning)

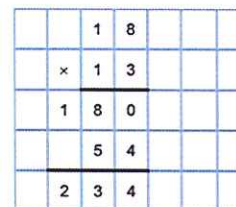
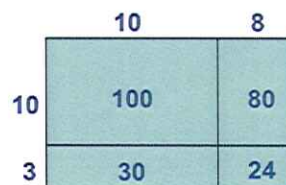
Solving practical problems where children need to scale up. Relate to known number facts.

Identify factor pairs for numbers

### Written methods (progressing to 4d x 2d)

Long multiplication using place value counters

Children to explore how the grid method supports an understanding of long multiplication (for  $2d \times 2d$ )



## Year 6

Continue with a range of equations as in Year 2 but with appropriate numbers. Also include equations with missing digits

### Mental methods

Identifying common factors and multiples of given numbers

Solving practical problems where children need to scale up. Relate to known number facts.

### Written methods

Continue to refine and deepen understanding of written methods including fluency for using long multiplication

X	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

$$\begin{array}{r}
 \begin{array}{cccc}
 & 2 & 3 & 1 \\
 & 1 & 3 & 4 & 2 \\
 \times & & 1 & 8 \\
 \hline
 1 & 3 & 4 & 2 & 0 \\
 1 & 0 & 7 & 3 & 6 \\
 \hline
 2 & 4 & 1 & 5 & 6 \\
 \hline
 & & & & 1
 \end{array}
 \end{array}$$



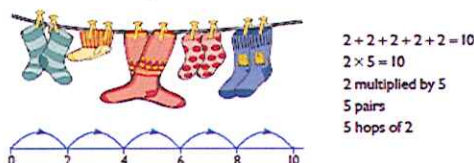
## Division

### Year 1

#### Mental Strategies

Children should experience [regular counting](#) on and back from different numbers in 1s and in multiples of 2, 5 and 10.

They should begin to recognise the number of groups counted to support understanding of relationship between multiplication and division.



Children should begin to understand division as both sharing and grouping.

Sharing – 6 sweets are shared between 2 people. How many do they have each?



Grouping-  
How many 2's are in 6?



They should use objects to group and share amounts to develop understanding of division in a practical sense. E.g. using Numicon to find out how many 5's are in 30? How many pairs of gloves if you have 12 gloves?

Children should begin to explore finding simple fractions of objects, numbers and quantities.

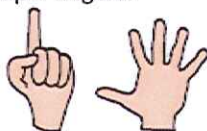
E.g. 16 children went to the park at the weekend. Half that number went swimming. How many children went swimming?

### Year 2

#### Mental Strategies

Children should count regularly, on and back, in steps of 2, 3, 5 and 10.

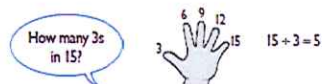
Children who are able to count in twos, threes, fives and tens can use this knowledge to work out other facts such as  $2 \times 6$ ,  $5 \times 4$ ,  $10 \times 9$ . Show the children how to hold out their fingers and count, touching each finger in turn. So for  $2 \times 6$  (six twos), hold up 6 fingers:



Touching the fingers in turn is a means of keeping track of how far the children have gone in creating a sequence of numbers. The physical action can later be visualised without any actual movement.

This can then be used to support finding out 'How many 3's are in 18?' and children count along fingers in 3's therefore making link between multiplication and division.

Children should continue to develop understanding of division as sharing **and** grouping.



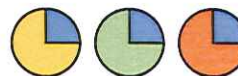
15 pencils shared between 3 pots, how many in each pot?

Children should be given opportunities to find a half, a quarter and a third of shapes, objects, numbers and quantities. Finding a fraction of a number of objects to be related to sharing.

They will explore visually and understand how some fractions are equivalent – e.g. two quarters is the same as one half.

[Use children's intuition to support understanding of fractions as an answer to a sharing problem.](#)

3 apples shared between 4 people =  $\frac{3}{4}$



### Year 3

#### Mental Strategies

Children should count regularly, on and back, in steps of 3, 4 and 8. Children are encouraged to use what they know about known times table facts to work out other times tables. This then helps them to make new connections (e.g. through doubling they make connections between the 2, 4 and 8 times tables).

Children will make use multiplication and division facts they know to make links with other facts.

$3 \times 2 = 6$ ,  $6 \div 3 = 2$ ,  $2 = 6 \div 3$

$30 \times 2 = 60$ ,  $60 \div 3 = 20$ ,  $2 = 60 \div 30$

They should be given opportunities to solve grouping and sharing problems practically (including where there is a remainder but the answer needs to be given as a whole number) e.g. Pencils are sold in packs of 10. How many packs will I need to buy for 24 children?

Children should be given the opportunity to further develop understanding of division (sharing) to be used to find a fraction of a quantity or measure.

[Use children's intuition to support understanding of fractions as an answer to a sharing problem.](#)

3 apples shared between 4 people =  $\frac{3}{4}$



#### Vocabulary

See Y1 and Y2  
inverse

#### Generalisations

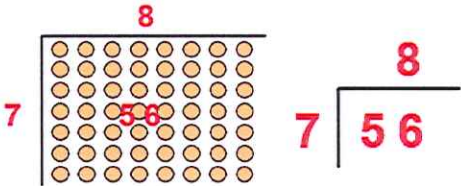
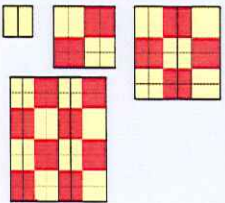
Inverses and related facts – develop fluency in finding related multiplication and division facts.

Develop the knowledge that the inverse relationship can be used as a checking method.

<p><b><u>Vocabulary</u></b> share, share equally, one each, two each..., group, groups of, lots of, array</p> <p><b><u>Generalisations</u></b></p> <ul style="list-style-type: none"> <li>• True or false? I can only halve even numbers.</li> <li>• Grouping and sharing are different types of problems. Some problems need solving by grouping and some by sharing. Encourage children to practically work out which they are doing.</li> </ul> <p><b><u>Some Key Questions</u></b> How many groups of...? How many in each group? Share... equally into... What can do you notice?</p>	<p><b><u>Vocabulary</u></b> group in pairs, 3s ... 10s etc equal groups of divide, ÷, divided by, divided into, remainder</p> <p><b><u>Generalisations</u></b> Noticing how counting in multiples of 2, 5 and 10 relates to the number of groups you have counted (introducing times tables)</p> <p>An understanding of the more you share between, the less each person will get (e.g. would you prefer to share these grapes between 2 people or 3 people? Why?)</p> <p>Secure understanding of grouping means you count the number of groups you have made. Whereas sharing means you count the number of objects in each group.</p> <p><b><u>Some Key Questions</u></b> How many 10s can you subtract from 60? I think of a number and double it. My answer is 8. What was my number? If <math>12 \times 2 = 24</math>, what is <math>24 \div 2</math>? Questions in the context of money and measures (e.g. how many 10p coins do I need to have 60p? How many 100ml cups will I need to reach 600ml?)</p>	<p><b><u>Some Key Questions</u></b> Questions in the context of money and measures that involve remainders (e.g. How many lengths of 10cm can I cut from 81cm of string? You have £54. How many £10 teddies can you buy?) What is the missing number? <math>17 = 5 \times 3 + \underline{\quad}</math> <math>\underline{\quad} = 2 \times 8 + 1</math></p>
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## Division

Year 4	Year 5	Year 6
<p><b><u>Mental Strategies</u></b> Children should experience regular counting on and back from different numbers in multiples of 6, 7, 9, 25 and 1000. Children should learn the multiplication facts to 12 x 12.</p> <p><b><u>Vocabulary</u></b> see years 1-3 divide, divided by, divisible by, divided into share between, groups of factor, factor pair, multiple times as (big, long, wide ...etc) equals, remainder, quotient, divisor inverse</p> <p><b><u>Towards a formal written method</u></b> Alongside pictorial representations and the use of models and images, children should progress onto short division using a bus stop method.</p>  <p>Place value counters can be used to support children apply their knowledge of grouping. Reference should be made to the value of each digit in the dividend.</p> <p><b><u>Each digit as a multiple of the divisor</u></b> 'How many groups of 3 are there in the hundreds column?' 'How many groups of 3 are there in the tens column?' 'How many groups of 3 are there in the units/ones column?'</p>	<p><b><u>Mental Strategies</u></b> Children should count regularly using a range of multiples, and powers of 10, 100 and 1000, building fluency. Children should practice and apply the multiplication facts to 12 x 12.</p> <p><b><u>Vocabulary</u></b> see year 4 common factors prime number, prime factors composite numbers short division square number cube number inverse power of</p> <p><b><u>Generalisations</u></b> The = sign means equality. Take it in turn to change one side of this equation, using multiplication and division, e.g. Start: <math>24 = 24</math> Player 1: <math>4 \times 6 = 24</math> Player 2: <math>4 \times 6 = 12 \times 2</math> Player 1: <math>48 \div 2 = 12 \times 2</math></p> <p><b><u>Sometimes, always, never true questions</u></b> about multiples and divisibility. E.g.:</p> <ul style="list-style-type: none"> <li>• If the last two digits of a number are divisible by 4, the number will be divisible by 4.</li> <li>• If the digital root of a number is 9, the number will be divisible by 9.</li> <li>• When you square an even number the result will be divisible by 4 (one example of 'proof' shown left)</li> </ul> 	<p><b><u>Mental Strategies</u></b> Children should count regularly, building on previous work in previous years. Children should practice and apply the multiplication facts to 12 x 12.</p> <p><b><u>Vocabulary</u></b> see years 4 and 5</p> <p><b><u>Generalisations</u></b> Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as PEMDAS, or could be encouraged to design their own ways of remembering.</p> <p>Sometimes, always, never true questions about multiples and divisibility. E.g.: If a number is divisible by 3 and 4, it will also be divisible by 12. (also see year 4 and 5, and the hyperlink from the Y5 column)</p> <p>Using what you know about <a href="#">rules of divisibility</a>, do you think 7919 is a prime number? Explain your answer.</p> <div data-bbox="1599 1027 2056 1331" style="border: 1px solid black; padding: 10px; margin-top: 20px;"> <p><b><u>Some Key Questions for Year 4 to 6</u></b></p> <p>What do you notice?</p> <p>What's the same? What's different?</p> <p>Can you convince me?</p> <p>How do you know?</p> </div>

$$\begin{array}{r} 112 \\ 3 \overline{) 336} \end{array}$$



When children have conceptual understanding and fluency using the bus stop method without remainders, they can then progress onto 'carrying' their remainder across to the next digit.

### Generalisations

True or false? Dividing by 10 is the same as dividing by 2 and then dividing by 5. Can you find any more rules like this?

Is it sometimes, always or never true that  $\square \div \Delta = \Delta \div \square$ ?

Inverses and deriving facts. 'Know one, get lots free!' e.g.:  $2 \times 3 = 6$ , so  $3 \times 2 = 6$ ,  $6 \div 2 = 3$ ,  $60 \div 20 = 3$ ,  $600 \div 3 = 200$  etc.

Sometimes, always, never true questions about multiples and divisibility. (When looking at the examples on this page, remember that they **may not** be 'always true'!) E.g.:

- Multiples of 5 end in 0 or 5.
- The digital root of a multiple of 3 will be 3, 6 or 9.
- The sum of 4 even numbers is divisible by 4.



## Year 1

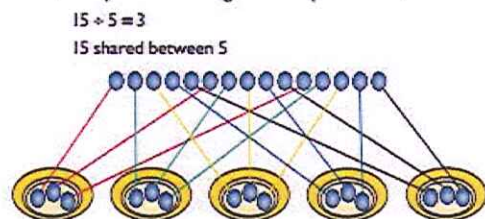
Children must have secure counting skills- being able to confidently count in 2s, 5s and 10s.

Children should be given opportunities to reason about what they notice in number patterns.

**Group AND share small quantities- understanding the difference between the two concepts.**

### Sharing

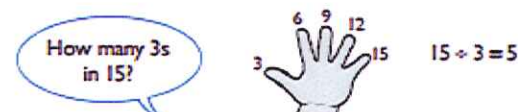
Develops importance of one-to-one correspondence.



Children should be taught to share using concrete apparatus.

### Grouping

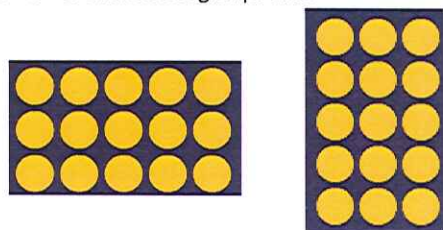
Children should apply their counting skills to develop some understanding of grouping.



Use of arrays as a pictorial representation for division.

$15 \div 3 = 5$  There are 5 groups of 3.

$15 \div 5 = 3$  There are 3 groups of 5.



Children should be able to find  $\frac{1}{2}$  and  $\frac{1}{4}$  and simple fractions of objects, numbers and quantities.

## Year 2

### $\div$ = signs and missing numbers

$$6 \div 2 = \square$$

$$\square = 6 \div 2$$

$$6 \div \square = 3$$

$$3 = 6 \div \square$$

$$\square \div 2 = 3$$

$$3 = \square \div 2$$

$$\square \div \nabla = 3$$

$$3 = \square \div \nabla$$

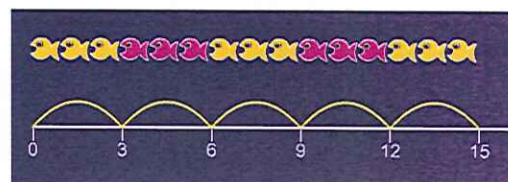
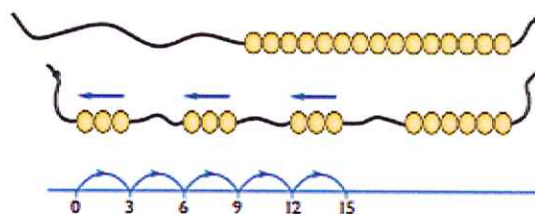
Know and understand sharing and grouping- introducing children to the  $\div$  sign.

Children should continue to use grouping and sharing for division using practical apparatus, arrays and pictorial representations.

### Grouping using a numberline

Group from zero in jumps of the divisor to find our 'how many groups of 3 are there in 15?'

$$15 \div 3 = 5$$



Continue work on arrays. Support children to understand how multiplication and division are inverse. Look at an array – what do you see?

## Year 3

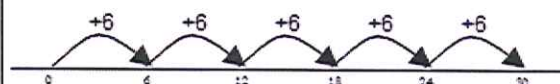
### $\div$ = signs and missing numbers

Continue using a range of equations as in year 2 but with appropriate numbers.

### Grouping

How many 6's are in 30?

$30 \div 6$  can be modelled as:



### Becoming more efficient using a numberline

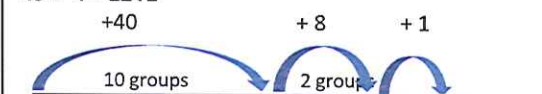
Children need to be able to partition the dividend in different ways.

$$48 \div 4 = 12$$



### Remainders

$$49 \div 4 = 12 \text{ r}1$$



Sharing – 49 shared between 4. How many left over?


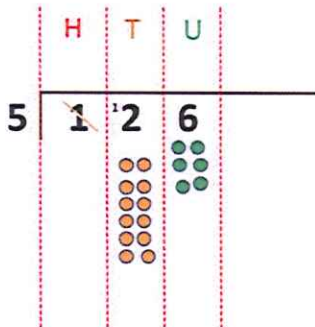
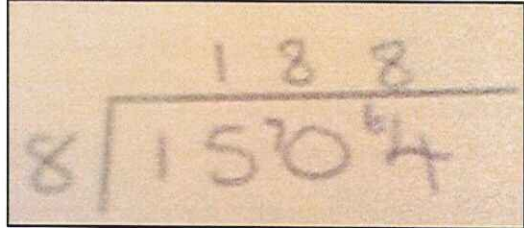
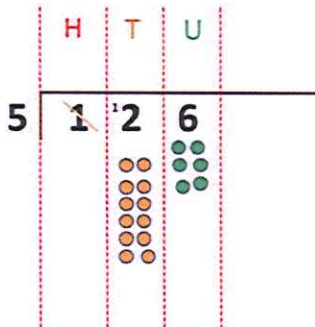
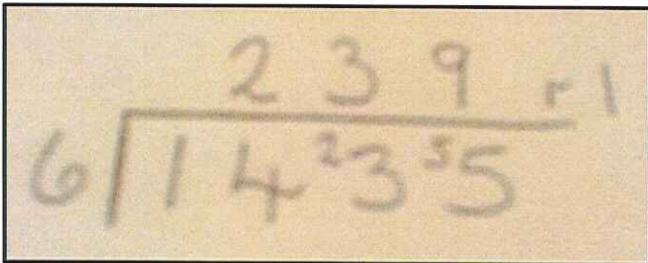
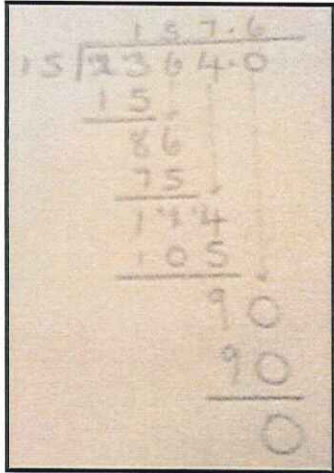
Grouping – How many 4s make 49. How many are left over?

Place value counters can be used to support children apply their knowledge of grouping.

For example:

$$60 \div 10 = \text{How many groups of 10 in 60?}$$

$$600 \div 100 = \text{How many groups of 100 in 600?}$$

Year 4	Year 5	Year 6
<p><u>÷ = signs and missing numbers</u> Continue using a range of equations as in year 3 but with appropriate numbers.</p> <p><u>Sharing, Grouping and using a number line</u> Children will continue to explore division as sharing and grouping, and to represent calculations on a number line until they have a secure understanding. Children should progress in their use of written division calculations:</p> <ul style="list-style-type: none"> <li>Using tables facts with which they are fluent</li> <li>Experiencing a logical progression in the numbers they use, for example:               <ol style="list-style-type: none"> <li>Dividend just over 10x the divisor, e.g. <math>84 \div 7</math></li> <li>Dividend just over 10x the divisor when the divisor is a teen number, e.g. <math>173 \div 15</math> (learning sensible strategies for calculations such as <math>102 \div 17</math>)</li> <li>Dividend over 100x the divisor, e.g. <math>840 \div 7</math></li> <li>Dividend over 20x the divisor, e.g. <math>168 \div 7</math></li> </ol> </li> </ul> <p>All of the above stages should include calculations with remainders as well as without. Remainders should be interpreted according to the context. (i.e. rounded up or down to relate to the answer to the problem)</p> <div data-bbox="741 544 1476 767"> <p>e.g. <math>840 \div 7 = 120</math></p> <p><i>Jottings</i>  <math>7 \times 100 = 700</math>  <math>7 \times 10 = 70</math>  <math>7 \times 20 = 140</math></p>  </div>	<p><u>Formal Written Methods</u> Formal short division should only be introduced once children have a good understanding of division, its links with multiplication and the idea of 'chunking up' to find a target number (see use of number lines above)</p> <p>Short division to be modelled for understanding using place value counters as shown below. Calculations with 2 and 3-digit dividends. E.g. fig 1</p> <div data-bbox="378 1078 692 1406">  </div>	<p><u>÷ = signs and missing numbers</u> Continue using a range of equations but with appropriate numbers</p> <p><u>Sharing and Grouping and using a number line</u> Children will continue to explore division as sharing and grouping, and to represent calculations on a number line as appropriate.</p> <p>Quotients should be expressed as decimals and fractions</p> <p><u>Formal Written Methods – long and short division</u> E.g. <math>1504 \div 8</math></p> <div data-bbox="1541 619 2063 847">  </div>
<p><u>Formal Written Methods</u> Formal short division should only be introduced once children have a good understanding of division, its links with multiplication and the idea of 'chunking up' to find a target number (see use of number lines above)</p> <p>Short division to be modelled for understanding using place value counters as shown below. Calculations with 2 and 3-digit dividends. E.g. fig 1</p> <div data-bbox="378 1078 692 1406">  </div>	<p><u>Formal Written Methods</u> Continued as shown in Year 4, leading to the efficient use of a formal method. The language of grouping to be used (see link from fig. 1 in Year 4) E.g. <math>1435 \div 6</math></p> <div data-bbox="846 983 1491 1246">  </div> <p>Children begin to practically develop their understanding of how express the remainder as a decimal or a fraction. Ensure practical understanding allows children to work through this (e.g. what could I do with this remaining 1? How could I share this between 6 as well?)</p>	<p>E.g. <math>2364 \div 15</math></p> <div data-bbox="1603 948 1935 1422">  </div>